SOAR Research Proposal – Summer 2019
Diameter Restricted Polyominoes and Optimal Network Reliability

Faculty: Nathan Shank, Associate Professor, Mathematics and Computer Science Department
Dates: June 3, 2019 - August 9, 2019 (10 weeks)
Students: Alvaro Belmonte, Junior, Mathematics Major, Graduate in Spring 2020
Rey Anaya, Sophomore, Mathematics Major, Graduate in Spring 2021.
Title: Diameter Restricted Polyominoes and Optimal Network Reliability

The Moravian College Mathematics Program recently received an National Science Foundation Research Experience for Undergraduate (NSF-REU) Site Grant. The three year grant provides summer funding for approximately 15 extremely talented undergraduate students from across the country to come to Moravian for a 9 week research experience in mathematics concurrent with SOAR. The NSF grant is intended to support students outside the home institution, however, if a qualified Moravian student can internal funding they can participate in the program.

Participating in the program has many benefits to Moravian and its students. Participating in such a program is a life-changing activity for students, Moravian students would have the opportunity to work with some of the brightest students from across the country, life-long associations develop in these programs, and students usually get at least one peer-reviewed publication. The program is centered around computational and experimental mathematics (CEM) which harnesses the power of computers to help further the boundary of mathematics. The re-emergence of the field has lead to some stimulating new research and we want to prepare our students to be at the forefront of these developments.

The program engages in discipline-appropriate scholarly research by using a cohort based model where each student is involved in two projects with 3-5 students and a faculty mentor. If funded, the SOAR students will be in at least one of the projects below with Dr. Shank as their mentor. Each cohort is required to give weekly presentations to the entire REU community which adds another layer of accountability, allows students to understand many different projects, and gives the students practice presenting their work. The SOAR students will satisfy all of the requirements of SOAR including working with a Moravian faculty, making a presentation to the SOAR community, working full time on their project, working with public relations, making a Scholars Day presentation, and writing a final report.

Description of the Project

Diameter Restricted Polyominoes: A polyomino is a two dimensional geometric figure made up of square cells that are joined along an edge. We classify the size of the polyomino by the number of cells, n. The classic game of Tetris is played with polyominoes of size $n = 4$. Investigations into polyominoes generally fall into two different categories: tiling and counting. Tiling problems involve finding ways to tile regions with polyominoes and counting problems involve counting the number of possible polyomino configurations based on the size n and other restrictions.

The Online Encyclopedia of Integer Sequences (OEIS) is a database of integer sequences containing over 320,000 sequences and over 700 sequence related to polyominoes including all polyominoes (OEIS A002212), free polyominoes (OEIS A000105), free polyominoes with holes (OEIS A001419), and fixed polyominoes (OEIS A001168). For small values of n, counting polyominoes can be done by hand, however, as n increases, the number of polyominoes increases to where hand computations are difficult if not impossible. For example, when $n = 20$, there are over **630 billion** polyominoes.

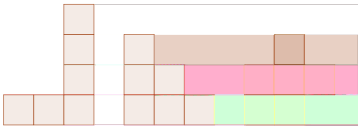

Figure 1: Examples of a polyominoes with $n = 6$ with diameters 5, 4, and 3.

In this project we want to count polyominoes of size n with a fixed diameter d. Here diameter is measures as the maximum number of squares traversed to get from one square to another. For example, if we are allowed to use $n = 20$ cells, how many polyominoes are possible if we restrict the diameter to $d = 10$? Finding the relationship between n and d would allow us to characterize polyominoes by diameter. When n is small, we can do this by hand, but as n increases, we will apply computational and experimental mathematics to help generate results and formulate conjectures.

Optimal Network Reliability: In mathematics we model a network as a set of vertices and a set of edges

will develop conjectures for larger, computational difficult, values of n. Ultimately we would like to be able to prove the conjecture for all values of n, however, at a minimum we will be able to contribute several new sequences to the OEIS.

TimeTable

Week	Objective
Week 1	Finish background reading and create presentations including summary of background reading and timeline.
Week 2-4	

SOAR Student Statement - Summer 2019
Diameter Restricted Polyominoes and Optimal Network Reliability

Faculty: Nathan Shank, Associate Professor, Mathematics and Computer Science Department
Dates: June 3, 2019 - August 9, 2019 (10 weeks)
Student: Alvaro Belmonte, Mathematics Major
Graduation: Spring 2020
Housing: Yes

Title: Diameter Restricted Polyominoes and Optimal Network Reliability

Rationale

"If you are the smartest person in the room, then you are in the wrong room." Attributed to no one, this quote has guided my intellectual ventures since I entered higher education. I have always tried to challenge myself by taking hard classes, trying to expand my abilities, and doing independent learning outside class. The main reason for doing this SOAR Project is to challenge myself and sharpen my intellect and mathematical tools that will help me in the future in graduate studies. This will challenge my knowledge that I have acquired during the last years of higher education, not only by directly applying what I have learned in classes, but by expanding the critical thinking skills developed during those years.

Participating in SOAR will help me develop, understand, and experience the process of mathematical research, where we try to

get a result of an unsolved problem and to understand why that solution actually works. The SOAR experience would not only enrich my summer intellectually, but also in the fall I will begin a year long Honors project and my experience in SOAR will help develop my research skills.

I am interested on both projects, so I am excited to work on both of them. I am interested in the Optimal Network Reliability project because when I was studying engineering, I was, and still am, really interested on power distribution and networks. Hence, I

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Dates: June 3, 2019 - August 9, 2019 (10 weeks)
Student: Rey Anaya, Mathematics Major & Computer Science Minor
Graduation: Spring 2021
Housing: Yes

Title: Diameter Restricted Polyominoes and Optimal Network Reliability

Rationale

Throughout high school and college, my favorite subject has been mathematics. Mathematics has always been interesting to me because of the precision involved, however, it is also one of the more difficult subjects for me to learn. The reason why I have been successful is due to the fact that I am dedicated and enjoy the time and eff

country. This is a very important step for me since my goal after Moravian College is to go to graduate school in mathematics and eventually become a mathematics professor. Although mathematics is thought to be done in isolation, in reality, people collaborate with peers often. Working in groups is encouraged, allows brainstorming of ideas, and allows us to further our research more efficiently. I am excited to build relationships with other REU students and faculty which I plan to continue after the su